ROBUSTNESS OF DISCRETE NONLINEAR SYSTEMS WITH OPEN-CLOSED-LOOP ITERATIVE LEARNING CONTROL

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Abstract:

In this paper, iterative learning control problem is investigated for a class of time-varying discrete nonlinear systems in the presence of different disturbances. Conditions for guaranteeing the robustness of open-closed-loop iterative learning control system are presented. It is shown that the tracking errors of the control system are uniformly bounded in the presence of bounded state uncertainty, output disturbance and initial state error. The system can track desired trajectory completely if all the uncertainties are gradually repeated. The conditions show that most coefficients of the iterative learning controller have no effects on the system robustness.

Keywords:

Discrete nonlinear system; Robustness; Learning control

1 Introduction

A common fact is that a person could study and improve his skills through repeated trials. This motivates the idea of iterative learning control (ILC) for systems performing repetitive tasks, e.g., robotic manipulators, disk drive systems, etc. Since it was first introduced by Arimoto (1984), ILC has received lots of attention because of its effectiveness and simplicity. Many papers have developed various ILC schemes. Moore (1998), Bien (1998), Xu (1998), Sun (1999), Xie (2000) gave good surveys on different ILC literature. Int. J. Control (Vol.73 No.10) even published a special issue on ILC in 2000. It is shown that most ILC researches have been focused on continuous-time, dynamic systems, and the development of ILC for discrete time systems has been restricted to linear systems (Wang, 1998). But for real-time implementation, all ILC schemes have to be designed in the discrete-time domain (Xu, 1997; Chien, 1998). On the other hand, as pointed out by Moon (1998), most existing literature has focused mainly on the derivation of sufficient conditions for the ILC system to converge; plant uncertainty has not been explicitly considered in the learning control design, though iterative learning controllers are usually adopted to obtain finer tracking accuracy under plant uncertainties.

ILC scheme is unlikely to be applied to real system without a feedback control (Moon, 1998). In ILC strategies, even though the learning gain matrices can be selected so that sufficient conditions for guaranteeing the convergence of the learning process are satisfied, tracking error can grow quite large before finally converging to zero (Hauser, 1987; Jang, 1995; Moon, 1998; Lee, 1997; etc.). This undesirable behavior can occur even when the applied learning algorithm has been proved to be exponentially convergent (Lee, 1997), because the learning control structure is basically open-loop (Jang, 1995). As is well known, feedback control is the most commonly used method to eliminate huge overshoot in system output, so it is quite natural to employ the current cycle error in ILC scheme to eliminate the huge overshoot.

Motivated by the above problems, we consider an open-closed-loop discrete ILC scheme for a class of uncertain nonlinear discrete systems in this paper. Sufficient conditions for guaranteeing the robustness of the ILC system are given based on a few assumptions. It is proven that the final tracking errors are bounded in the presence of uncertainty, disturbance and initialization error under certain conditions. The paper is organized as follows. In section 2, we state the ILC problem. Section 3 contains the main result concerning the robustness issue of the ILC system. And finally, the conclusion appears in section 4.

2 Problem Formulation

Consider a class of discrete time-varying nonlinear systems that perform a given task repeatedly on a finite time interval $[0,N]$ (N is a positive integer). The systems can be described by the following different equations:

$$\begin{cases} x_k(t+1)=f(t,x_k(t),u_k(t))+\psi_k(t,x_k(t),u_k(t)) \\ y_k(t)=g(t,x_k(t))+D(t)u_k(t)+\eta_k(t,x_k(t),u_k(t)) \end{cases} \qquad (1)$$

where k indicates the number of operation cycle, t is the discrete time index and $t\in[0,N]$, state vector $x_k(t)\in R^n$, input vector $u_k(t)\in R^m$, output vector $y_k(t)\in R^r$, $f(\bullet,\bullet,\bullet)$: $R^n\times R^m\times[0,N]\to R^n$, $g(\bullet,\bullet)$: $R^n\times[0,N]\to R^r$, $D(\bullet)$: $R^n\times[0,N]\to R^{r\times m}$ with bounds as $\|D(\bullet)\|\le B_D$, state disturbance vector $\psi_k(\bullet,\bullet,\bullet)$: $R^n\times R^m\times[0,N]\to R^n$, output disturbance vector $\eta_k(\bullet,\bullet,\bullet)$: $R^n\times R^m\times[0,N]\to R^n$, for all

$t\in[0,N]$. The vector functions f and g are globally uniformly Lipschitz in x and u on $[0,N]$ in the sense of

$$\begin{cases}\|f(t,x_1,u_1)-f(t,x_2,u_2)\|\le f_0(\|x_1-x_2\|+\|u_1-u_2\|)\\ \|g(t,x_1)-g(t,x_2)\|\le g_0\|x_1-x_2\|\end{cases} \tag{2}$$

$$\forall x_1,x_2,u_1,u_2, \text{for} \quad t\in[0,N]$$

where f_0 and g_0 are positive constants. Disturbances $\psi_k(t, x_k(t), u_k(t))$、$\eta_k(t, x_k(t), u_k(t))$ and initial state $x_k(0)$ satisfy:

$$\begin{cases}\|\psi_{k+1}(t,x_{k+1}(t),u_{k+1}(t))-\psi_k(t,x_{k+1}(t),u_{k+1}(t))\|\le d_\psi\\ \|\eta_{k+1}(t,x_{k+1}(t),u_{k+1}(t))-\eta_k(t,x_k(t),u_k(t))\|\le d_\eta\\ \|x_{k+1}(0)-x_k(0)\|\le d_0\end{cases} \tag{3}$$

where d_ψ, d_η and d_0 are positive constants. We want the system output $y(t)$ to track a given trajectory $y_d(t)$ for all $t\in[0,N]$. Let us consider an open-closed-loop iterative learning control update law:

$$u_{k+1}(t)=u_k(t)+\sum_{j=0}^{t}L_{j,k}(t,x_k(t))e_k(t-j)+\sum_{j=0}^{t}B_{j,k+1}(t,x_{k+1}(t))e_{k+1}(t-j) \tag{4}$$

where $e_k(t)=y_d(t)-y_k(t)$ is the tracking error; $L_{j,k}(\bullet,\bullet)$、$B_{j,k}(\bullet,\bullet)$: $R^n\times[0,N]\to R^{r\times m}$ are bounded learning gain matrices, i.e.: $\|L_{j,k}(\bullet,\bullet)\|\le B_L$, $\|B_{j,k}(\bullet,\bullet)\|\le B_B$,and $[I+D(t)B_{0,k+1}(j, x_{k+1}(j)]^{-1}$ is existing and bounded, i.e.: $\|[I+D(t)B_{0,k+1}(j, x_{k+1}(j)]^{-1}\le B_I$。 The main results will be given in next section.

3 Main result

Theorem. Consider the discrete control system described by equations (1) and (4), if the following inequality:

$$\|[I+D(t)B_{0,k+1}(t,x_{k+1}(t))]^{-1}[I-D(t)L_{0,k}(t,x_k(t))]\|\le\rho<1 \tag{5}$$

holds for all x and t, then the tracking error will be bounded when $k\to\infty$ as specified in (17); if all disturbances become repetitive gradually, then $y_k(t)$ will converge to $y_d(t)$ when $k\to\infty$。

Proof of Theorem

From equations (1) and (4) , we have:

$$\begin{aligned}e_{k+1}(t)-e_k(t)=&g(t,x_k(t))-g(t,x_{k+1}(t))+\\&\eta_k(t,x_k(t),u_k(t))-\\&\eta_{k+1}(t,x_{k+1}(t),u_{k+1}(t))-\\&D(t)[\sum_{j=0}^{t}L_{j,k}(t,x_k(t))e_k(t-j)+\\&\sum_{j=0}^{t}B_{j,k+1}(t,x_{k+1}(t))e_{k+1}(t-j)\ ]\end{aligned} \tag{6}$$

$$\begin{aligned}i.e:\ [I+D(t)B_{0,k+1}(t,x_{k+1}(t))]e_{k+1}(t)=&\\ [I-D(t)L_{0,k}(t,x_k(t))]e_k(t)+&\\ g(t,x_k(t))-g(t,x_{k+1}(t))+&\\ \eta_k(t,x_k(t),u_k(t))-&\\ \eta_{k+1}(t,x_{k+1}(t),u_{k+1}(t))-&\\ D(t)\begin{pmatrix}\sum_{j=1}^{t}L_{j,k}(t,x_k(t))e_k(t-j)+\\ \sum_{j=1}^{t}B_{j,k+1}(t,x_{k+1}(t))e_{k+1}(t-j)\end{pmatrix}&\end{aligned} \tag{7}$$

According to inequality (4), multiple $[I+D(t)\ B_{0,k+1}(0, x_{k+1}(0)]^{-1}$ on both sides of equation (7) and take norms on both sides, we get:

$$\begin{aligned}\|e_{k+1}(t)\|\le&\rho\|e_k(t)\|+m_1g_0\|x_{k+1}(t)-x_k(t)\|+m_1d_\eta+\\&m_1B_Dm_2\sum_{j=1}^{t}(\|e_k(t-j)\|+\|e_{k+1}(t-j)\|)\\=&\rho\|e_k(t)\|+m_1g_0\|x_{k+1}(t)-x_k(t)\|+m_1d_\eta+\\&m_1B_Dm_2\sum_{j=0}^{t-1}(\|e_k(j)\|+\|e_{k+1}(j)\|)\end{aligned} \tag{8}$$

$$\text{where } m_1=\sup_{t\in[0,T]}\|[I+D(t)B_{0,k+1}(t,x_{k+1}(t))]^{-1}\|$$

$$m_2=\max(B_L,B_B)$$

From equation (1), we have:

$$\begin{aligned}&\|x_{k+1}(t)-x_k(t)\|\\&=\| f(t-1,x_{k+1}(t-1),u_{k+1}(t-1))-\\&\quad f(t-1,x_k(t-1),u_k(t-1))+\\&\quad \psi_{k+1}(t-1,x_{k+1}(t-1),u_{k+1}(t-1))-\\&\quad \psi_k(t-1,,x_{k+1}(t-1),u_{k+1}(t-1))\|\\&\le f_0\|x_{k+1}(t-1)-x_k(t-1)\|+\\&\quad u_0\|u_{k+1}(t-1)-u_k(t-1)\|+d_\psi \qquad (9)\end{aligned}$$

By equation (4):

$$\begin{aligned}&\|u_{k+1}(t)-u_k(t)\|\\&=\|\sum_{j=0}^{t}L_{j,k}(t,x_k(t))e_k(t-j)+\\&\quad\sum_{j=0}^{t}B_{j,k+1}(t,x_{k+1}(t))e_{k+1}(t-j)\|\\&\le m_2\sum_{j=0}^{t}(\|e_k(j)\|+\|e_{k+1}(j)\|) \qquad (10)\end{aligned}$$

Combine inequalities (9) and (10), we get:

$$\begin{aligned}&\|x_{k+1}(t)-x_k(t)\|\\&\le f_0\|x_{k+1}(t-1)-x_k(t-1)\|+\\&\quad u_0m_2\sum_{j=0}^{t-1}(\|e_k(j)\|+\|e_{k+1}(j)\|)+d_\psi\\&\le f_0^2\|x_{k+1}(t-2)-x_k(t-2)\|+(f_0+1)d_\psi\\&\quad (f_0+1)u_0m_2\sum_{j=0}^{t-1}(\|e_k(j)\|+\|e_{k+1}(j)\|)\\&\le\cdots\\&\le f_0^t\|x_{k+1}(0)-x_k(0)\|+\\&\quad (f_0^{t-1}+f_0^{t-2}+\cdots+f_0^2+f_0+1)*\\&\quad u_0m_2\sum_{j=0}^{t-1}(\|e_k(j)\|+\|e_{k+1}(j)\|)+\\&\quad (f_0^{t-1}+f_0^{t-2}+\cdots+f_0^2+f_0+1)d_\psi\\&\le m_3+m_4\sum_{j=0}^{t-1}(\|e_k(j)\|+\|e_{k+1}(j)\|) \qquad (11)\end{aligned}$$

where

$$m_3=f_0^td_0+(f_0^{t-1}+f_0^{t-2}+\cdots+f_0^2+f_0+1)d_\psi$$

$$m_4=(f_0^{t-1}+f_0^{t-2}+\cdots+f_0^2+f_0+1)u_0m_2$$

By inequalities (8) and (11), we have:

$$\begin{aligned}&\|e_{k+1}(t)\|\\&\le\rho\|e_k(t)\|+m_1g_0m_3+m_1d_\eta+\\&\quad (m_1g_0m_4+m_1B_Dm_2)\sum_{j=0}^{t-1}(\|e_k(j)\|+\|e_{k+1}(j)\|)\\&=\rho\|e_k(t)\|+m_5+m_6\sum_{j=0}^{t-1}(\|e_k(j)\|+\|e_{k+1}(j)\|)\end{aligned} \qquad (12)$$

Where $m_5=m_1g_0m_3+m_1d_\eta$,

$m_6=m_1g_0m_4+m_1B_Dm_2$

According to the definition of λ- norm, $\|e_k(t)\|_\lambda=\sup_{t\in[0,T]}e^{-\lambda t}\|e_k(t)\|$, we have:

$$\begin{aligned}&e^{-\lambda t}\sum_{j=0}^{t-1}\|e_k(j)\|\\&=e^{-\lambda t}e^0\|e_k(0)\|+e^{-\lambda(t-1)}e^{-\lambda}\|e_k(1)\|+\\&\quad e^{-\lambda(t-2)}e^{-2\lambda}\|e_k(2)\|+\cdots+\\&\quad e^{-\lambda}e^{-\lambda(t-1)}\|e_k(t-1)\|\\&\le(e^{-\lambda t}+e^{-\lambda(t-1)}+\cdots+e^{-\lambda})\|e_k(t)\|_\lambda\\&\le te^{-\lambda}\|e_k(t)\|_\lambda \quad \text{for } t\ge1,\ \lambda>0 \qquad (13)\end{aligned}$$

According to inequality (13), multiple $e^{-\lambda t}$ to both sides of inequality (12) and takeλ-norm on both sides, we get:

$$\|e_{k+1}(t)\|_\lambda\le\rho\|e_k(t)\|_\lambda+m_5+m_6te^{-\lambda}\|e_k(t)\|_\lambda+ m_6te^{-\lambda}\|e_{k+1}(t)\|_\lambda \qquad (14)$$

Ifλ is big enough, from inequality (14) we have:

$$\|e_{k+1}(t)\|_\lambda\le\rho_1\|e_k(t)\|_\lambda+M \qquad (15)$$

where $M=\dfrac{m_5}{1-m_6te^{-\lambda}}$, $\rho_1=\dfrac{\rho+m_6te^{-\lambda}}{1-m_6te^{-\lambda}}$,

and $M>0,\ 0<\rho_1<1$。

From inequality (15) we have:

$$\|e_{k+1}(t)\|_\lambda \le \rho_1^2 \|e_{k-1}(t)\|_\lambda + \frac{1-\rho_1^2}{1-\rho_1} M$$
$$\le \cdots$$
$$\le \rho_1^{k-J+1} \|e_J(t)\|_\lambda + \frac{1-\rho_1^{k-J+1}}{1-\rho_1} M \quad (16)$$

i.e.: $\|e_k(t)\|_\lambda \to \dfrac{M}{1-\rho_1}$ when $k \to \infty$ (17)

If all disturbancs become reperitivegradually,

i.e., $d_\psi \to 0, d_n \to 0, d_0 \to 0$ when $k \to \infty$,

then $M \to 0$, $\|e_k(t)\|_\lambda \to 0$,

so $y_k(t) \to y_d(t)$ when $k \to \infty$。

Remark 1. Inequality (16) shows that system robustness can be ensured even when the conditions (4) are not guaranteed during the previous J iterations. Here J is a finite positive integer.

Remark 2. The theorem shows that learning gain matrices $B_{j,k}(i,x_k(i))$ (j>0) and $L_{j,k}(i,x_k(i))$ (j>0) have no influence on system robustness if they are bounded. The robustness condition is independent of function f(•,•,•) if the functions are globally uniformly Lipschitz in x and u on [0,N].

4 Conclusion

In this paper, the robustness of open-closed-loop iterative learning control is investigated for a class of time-varying discrete nonlinear systems in the presence of bounded state uncertainty, output disturbance and initial state error. It is shown that the convergence of output error within a bound can be ensured in the presence of above disturbances under some conditions. Furthermore, if these disturbances tend to be repetitive gradually, the convergence of output error can be ensured to reduce to zero. The conditions show that most coefficients of the iterative learning control law have no effects on the system robustness. The results here mean that: when you design an iterative leaning control system described by (1) and (4), repetitive disturbances can be ignored; by choosing learning matrices carefully to make ρ and M become smaller, we could get a faster convergent rate.

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